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Making Reasoning Visible: Cognitive Interview Evidence for Numeric Relational Reasoning Learning Progressions for Grades K-2

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Promising Math

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MMaRS Background



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Measuring Early Mathematical Reasoning Skills (MMaRS) Project

Purpose:

Develop formative assessment tools for teachers to use in K-2 to measure and monitor progress with **numeric relational reasoning** and spatial reasoning

Current Activities

- Developing item models
- Conducting Think-Alouds with K-2 students



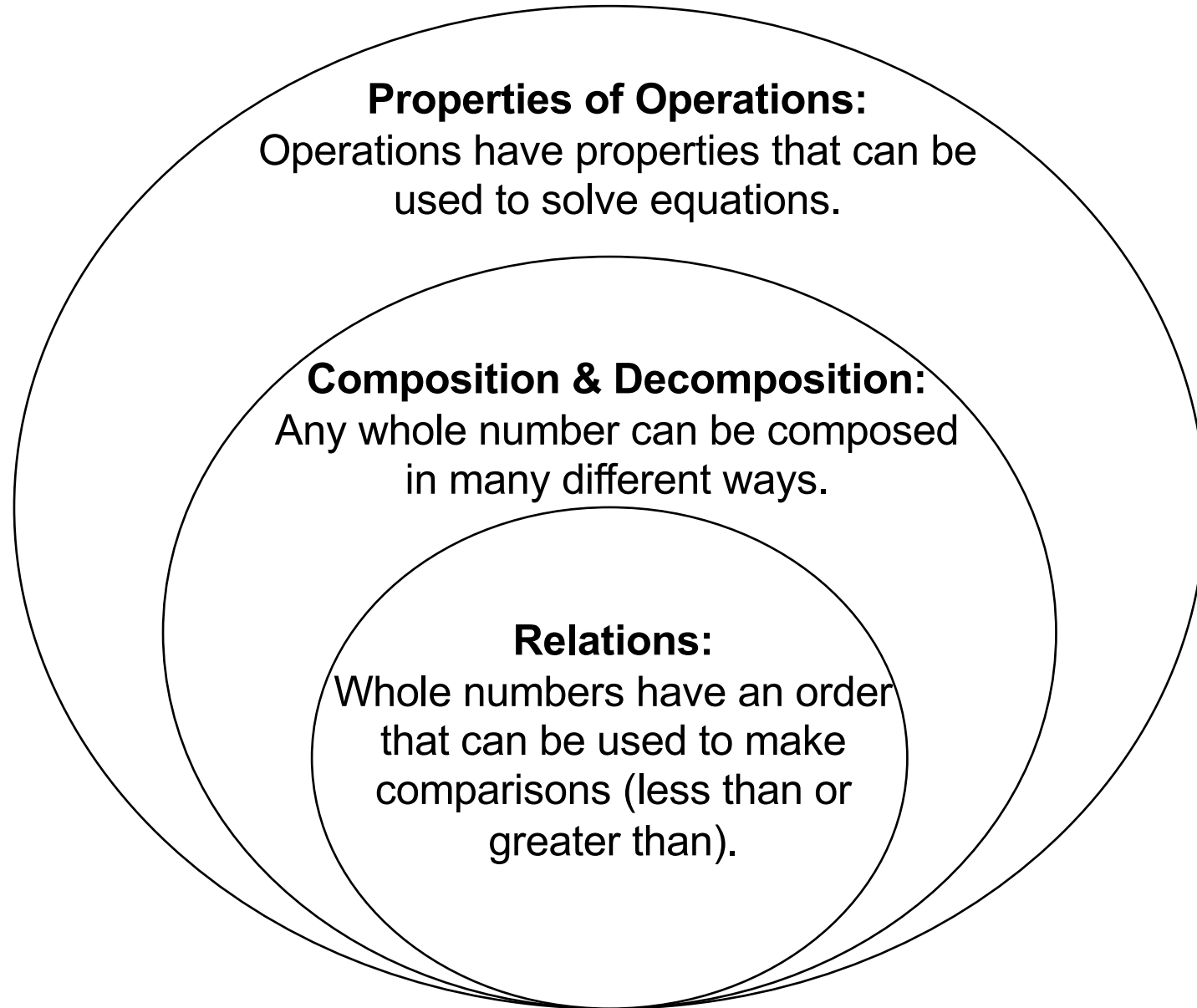
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#MMaRS



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Numeric Relational Reasoning



Presentation Outline

- MMaRS Background
- Cognitive Interview Example
- Numeric Relational Reasoning
- Learning Progression
- Methods
- Findings
- Future Activities





Cognitive Interview

Sample Task



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A woman with dark hair, wearing a black top and a name tag, is seated at a table, looking down at small white cards in her hands. A young girl with long brown hair, wearing a white shirt and a dark plaid vest, is seated next to her, looking at a large binder on the table. The table is covered with various papers, including a large binder with a colorful tab, and a purple marker. In the background, there is a green chalkboard with a decorative border and a cardboard box with the number '48' and the word 'Criss' on it.

Bridging Research and Practice



Equation Card Sort

Turn
and
TALK

Pre K - K	True	?	Not True
K - 1	$a = a$ 12/32 (37.5%)		$a + b = c$ 19/32 (59%)
1 - 2	$a + b = c + d$ 7/32 (22%)		$c = a + b$ 10/32 (31%)
	$a + b = b + a$ 4/32 (12.25%)		



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Recognize true and not true equations with known numbers using **one or more property of operations.**



$$16 + 17 = 17 + 16$$

EK: What do you think?

Daisy: I'm not sure I think.

EK: You're not sure?

Daisy: *What does the equal sign really even mean in the middle?*

EK: So that's a good question. What do you think it means?

Daisy: Maybe if these two are the same as these two?

EK: Okay. So if it meant that, if it means ... If the equal sign in the middle means that this stuff on this side is the same as this stuff on this side, what would you think about this number sentence?

Daisy: That they're the same.

EK: That they're the same. Awesome. That's some awesome thinking.

[I was so excited that she was making the connection, my questioning skills went away temporarily!]



Numeric Relational Reasoning

Learning Progressions

Definitions



Numeric Relational Reasoning

- *Relational reasoning:*

“ability to recognize or derive meaningful relations between and among pieces of information that would otherwise be unrelated”

(Dumas, Alexander, & Grossnickle, 2013, p. 392)

- *Numeric relational reasoning:*

ability to mentally analyze relationships between numbers or expressions, often using knowledge of properties of operations, decomposition, and known facts

(Baroody, Purpura, Eiland, Reid, & Paliwal, 2016; Carpenter, Franke, & Levi, 2003; Farrington-Flint, Canobi, Wood, & Faulkner, 2007; Jacobs, Franke, Carpenter, Levi, & Battey, 2007)

$$15 + 28 = \underline{\quad} + 15 \qquad 22 + 13 = 10 + \underline{\quad} \qquad \underline{\quad} = 8 + 7$$



Numeric Relational Reasoning

Properties of Operations:

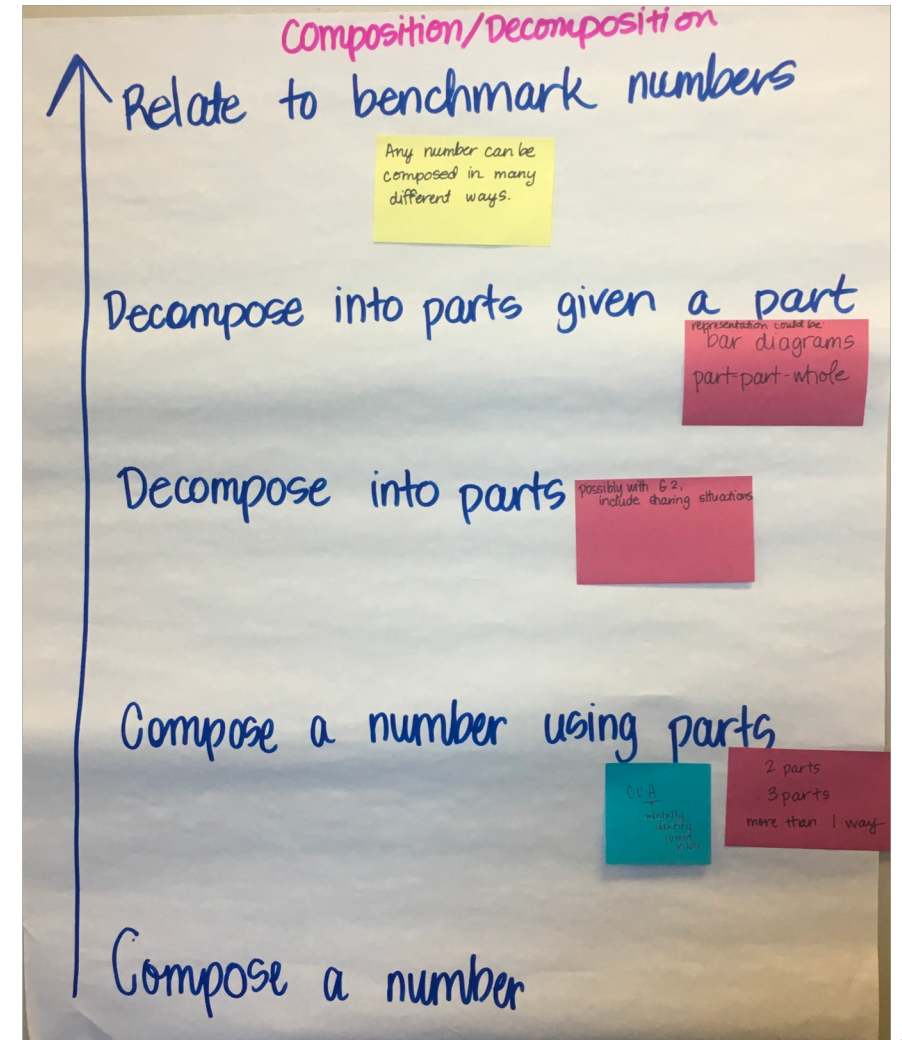
Operations have properties that can be used to solve equations.

Composition & Decomposition:

Any whole number can be composed in many different ways.

Relations:

Whole numbers have an order that can be used to make comparisons (less than or greater than).



Numeric Relational Reasoning (Draft as of November 2018)

Relations

Whole numbers have an order that can be used to make comparisons (less than or greater than).

1. Comparison

7 skills

2. Numbers Have Order

2 skills

3. Transitivity

6 skills

4. Representations

6 skills

Composition & Decomposition

Any whole number can be composed in many different ways.

5. Composition

5 skills

6. Decomposition

6 skills

7. Applying and Representing Composition and Decomposition

5 skills

Properties of Operations

Operations have properties that can be used to solve equations.

8. Equivalence of Quantity and Number

7 skills

9. Equal Sign as a Relational Symbol

4 skills

10. Maintaining Equality

4 skills

11. Solving for Unknown Values

7 skills

Learning Progressions (Draft as of November 2018)

“descriptions of successfully more sophisticated ways of reasoning within a content domain” (Smith, Wiser, Anderson, & Krajcik, 2006, p. 1)

Grade Band

Core Concept

	5. Composition								
Code	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
NRR.B.5.a.	Compose a number with single objects.								
NRR.B.5.b.		Compose a number with two parts.							
NRR.B.5.c.				Compose a number with three or more parts.					
NRR.B.5.d.				Compose a number with two or more parts using different number combinations .					
NRR.B.5.e.				Compose a number with two or more parts using concepts of place value .					

Essentialized
Skill Statements



Elements That Vary: Number Ranges

Foundational
Bridging
Target

	Kindergarten				Grade 1			Grade 2		
	F	B	T		F	B		F	B	T
Number/ Quantity	5	10	19	19	99	99	99	999	999	999

5. Composition									
Code	Kindergarten				Grade 1		Grade 2		
	F	B	T	F	B	T	F	B	T
NRR.B.5.a.	Compose a number with single objects.								
NRR.B.5.b.		Compose a number with two parts.							
NRR.B.5.c.			Compose a number with three or more parts.						
NRR.B.5.d.			Compose a number with two or more parts using different number combinations.						
NRR.B.5.e.			Compose a number with two or more parts using concepts of place value.						





Methods



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Research Question

Properties of Operations:

Operations have properties that can be used to solve equations.

Composition & Decomposition:

Any whole number can be composed in many different ways.

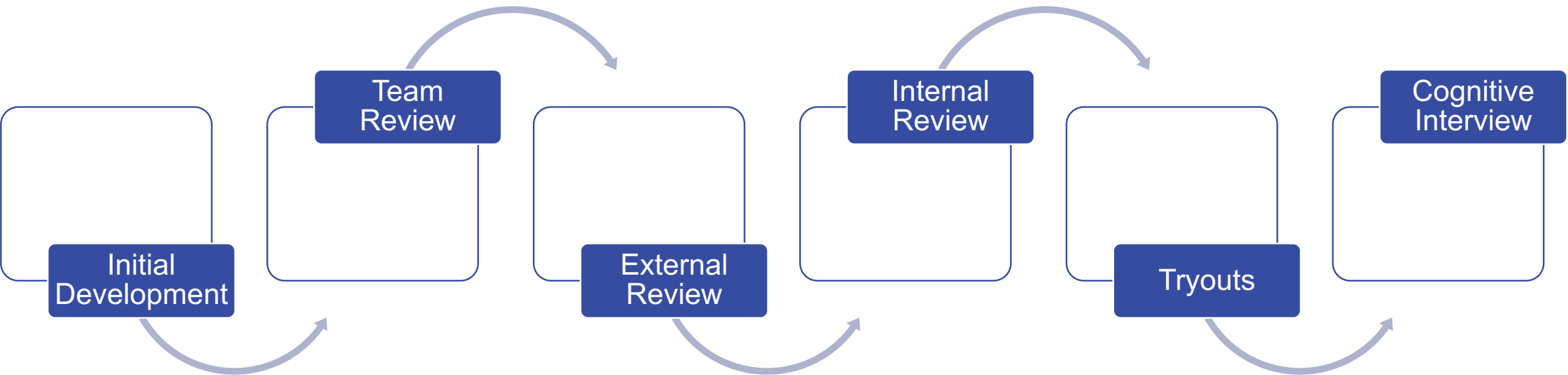
Relations:

Whole numbers have an order that can be used to make comparisons (less than or greater than).

What are characteristics of kindergarten, first- and second-grade students' Numeric Relational Reasoning within one-on-one cognitive interviews?



Learning Progression Revision Process



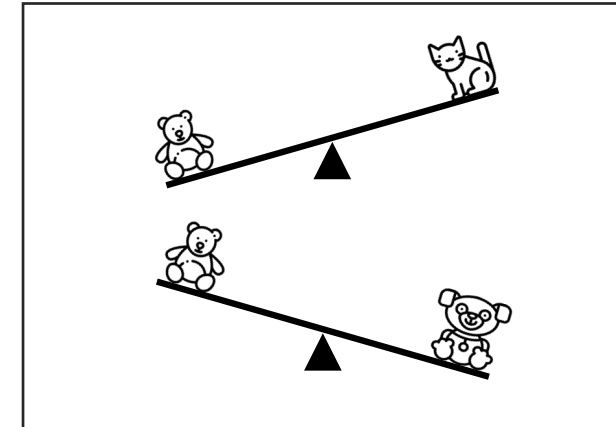
5. Composition									
Code	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
NRR.B.5.a.	Compose a number with single objects.								
NRR.B.5.b.				Compose a number with two parts.					
NRR.B.5.c.				Compose a number with three or more parts.					
NRR.B.5.d.				Compose a number with two or more parts using different number combinations.					
NRR.B.5.e.				Compose a number with two or more parts using concepts of place value.					

5. Composition									
Code	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
NRR.B.5.a.	Compose a number with single objects.								
NRR.B.5.b.				Compose a number with two parts.					
NRR.B.5.c.				Compose a number with three or more parts.					
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NRR.B.5.e.				Compose a number with two or more parts using concepts of place value.					

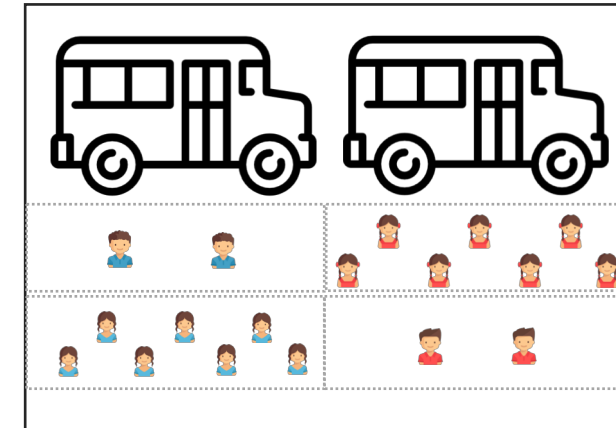


Cognitive Interview Development

Order three unspecified weights using balances.



Given a contextual situation with known quantities, use one or more **property of operations** to recognize when equality is maintained.

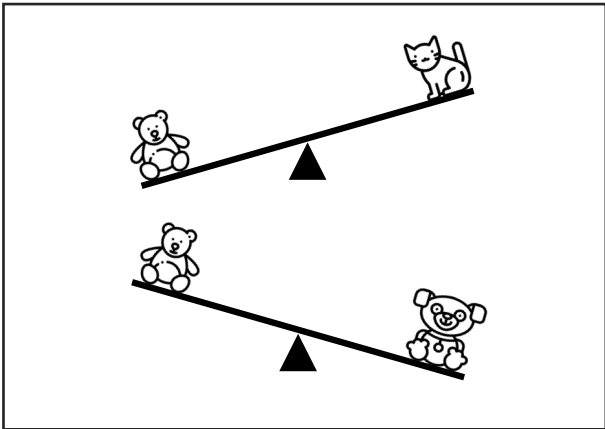


Recognize true and not true equations with known numbers using one or more **property of operations**.

True	?	Not True



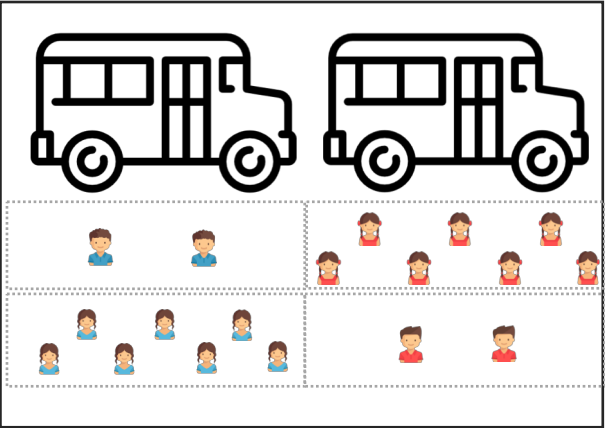
Content and Reasoning Questions



C1: Which animal weighs the most?

C2: Which animal weighs the least?

R: Ben thinks the bear is the heaviest item. How would you help him understand if he is right or wrong?



C: Julie thinks that more students rode on this bus Since more students got on first. Are there the same, more, or less number of students in the buses?

R: Can you show me how you know that there are the [same, more, or less]?

True	?	Not True

C: Is this card true, not true, or are you not sure?

R: What can you tell me about this card? Why did you put it here?



Participants & Sampling

Participant sample by school, grade, and support level as identified by classroom teacher.

Kindergarten				First Grade			Second Grade			Third Grade		
Support Level												
School												

Data Coding Example

EK: What do you think?

Daisy: I'm not sure I think.

EK: You're not sure?

Daisy: What does the equal sign really even mean in the middle?

EK: So that's a good question. What do you think it means?

Daisy: Maybe if these two are the same as these two?

EK: Okay. So if it meant that, if it means ... If the equal sign in the middle means that this stuff on this side is the same as this stuff on this side, what would you think about this number sentence?

Daisy: That they're the same.

EK: That they're the same. Awesome. That's some awesome thinking.

$$16 + 17 = 17 + 16$$

Recognize true and not true equations with known numbers using **one or more property of operations**.

Question equal sign (no strategy)

A: Compare numbers →
Relational meaning of equal sign

A: **Relational** meaning of equal sign





Findings: Anticipated and Unanticipated Strategies



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NRR.10.d. Summary of Anticipated and **Unanticipated** Strategies

Recognize true and not true equations with known numbers using **one or more** **property of operations**.

Gr	0-5	0-10	0-19	0-50	0-99	0-199
K	NME (2)	Unknown Equation Structure (1) NME (1)				
1		NME (2)	Operational, Unknown Equation Structure (2)			
2				Relational (1) 8c (1) Compare Sums (1)	Operational (2)	Unknown Equation Structure (1)

NME: No Mathematical Evidence



10. Maintaining Equality									
Code	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
NRR.C.10.d.						Recognize true and not true equations with known numbers using one or more properties of operations.			

Unanticipated Strategy

Examples

Unknown Equation Structure.

[76 + 87 = 87 + 76] So I don't really know what's going on because ... I wanna know why they put these in different ... like why did they swap them? (Donna, 24:43).

It's again they have the same numbers and it's confusing that they are just different sections. 'Cause I do not think that $134 - 56 + 56 = 134$ (Donna, 25:43).

Compare Sums. [17 + 12 + 12 = 24 + 12]

36 for these two and 41 here and ... I forgot what I said for this one ... 36 ... They're not true (Daisy, 40:45).

Unanticipated strategies: NRR.C.9.a-d

Skill Code			C.9.a		C.9.b	C.9.c				C.9.d		
Unanticipated			C	NME	C	C a+b=c	C a=a	C c=a+b	Unknown Equation Structure	C a+b=c+d	C a+a=b+b	Unknown Equation Structure
SID	Gr.	Number Range										
Neil	K	0 - 5	1	-	-	-	1	-	-	-	-	-
Nadia	K	0 - 5	1	1	-	-	1	1	-	1	-	-
Nicole	K	0 - 10	1	-	-	1	-	-	-	-	-	-
Noah	K	0 - 10	1	-	-	-	-	-	-	-	1	-
Tracy	1	0 - 10	1	-	-	1	1	-	a=a	-	-	-
Thomas	1	0 - 10	1	-	-	-	-	-	a=a	-	-	-
Terri	1	0 - 19	1	-	-	1	-	1	a=a	1	-	a+a = b+b
Tasha	1	0 - 19	1	-	-	1	-	-	a=a	-	1	a+b = c+d
Daisy	2	0 - 50	1	-	-	1	-	1	a=a	-	-	a+b = c+d; a+a = b+b
Divya	2	0 - 99	1	-	-	1	1	1	-	-	1	-
Demarcus	2	0 - 99	1	-	-	1	1	1	-	1	1	-
Donna	2	0 - 199	1	-	1	1	1	-	-	-	-	a+b = c+d
Total			12	1	1	8	6	5	5	3	4	4

8. Equivalence of Quantity and Number									
Code	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
NRR.C.8.f.		Recognize two equivalent expressions that demonstrate one or more property of operations.							

Unanticipated Strategy	Examples
Compare Number of Addends.	<i>Because these numbers $[2 + 2 + 1]$ only have three, and these numbers $[1 + 4]$ only have two (Neil, 14:24).</i>
Blocks.	<i>I'll show them by using blocks, if I had some. And telling her which numbers are the same because she has to know her numbers to know. So, I would get the blocks and get six and tell her the number sentences (Divya, 16:35).</i>

Findings: Composition and Decomposition

5. Composition									
Code	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
NRR.B.5.a.	Compose a number with single objects.								
NRR.B.5.b.		Compose a number with two parts.							
NRR.B.5.c.			Compose a number with three or more parts.						
NRR.B.5.d.			Compose a number with two or more parts using different number combinations .						
NRR.B.5.e.			Compose a number with two or more parts using concepts of place value .						

Skill Code			B.5.a.		B.5.b.			B.5.c.				B.5.d.			B.5.e.	
Unanticipated Strategies			C	Counting	C	Counting	Writing	C	Construc ting Numbers Using Digits	Counting	Writing	C	Compare Numbers to Find Same	Counting	C	NME
SID	Grade	Number Range														
CK	K	0-5	1	-	0	-	-	0	1	-	-	0	-	-	[0-19] 0	-
341	K		1	-	1	-	Expression	1	-	-	Expression	1	1	-	[0-19] 0	-
708	1	0-10	1	Parts & Combine Groups	0	Parts & Combine Groups	-	1	-	Parts & Combine Groups	-	1	-	Parts & Combine Groups	[0-19] 0	-
769	1		1	Skip (10)	1	-	Equation	1	-	-	Equation	1	-	-	[0-19] 0	-
223	1	0-19	1	Skip (10)	1	-	-	1	-	Parts & Combine Groups	-	1	-	-	[0-19] 0	-
			1	-	-	-	-	1	-	-	-	1	-	1	0	1
352	2	0-50	1	Mental	[0-199] 1	-	-	[0-199] 1	-	-	-	[0-199] 1	1	-	[0-199] 1	1
Strategies Total by Skill Code			6	4	4	1	2	5	1	2	2	5	2	1	1	2



Findings: Relations

Unanticipated Strategy. Justification/Description	Examples
Value of digits. Children compared the value of digits within numbers without explicitly using place value. Children correctly aligned numbers in the tens and ones places when comparing without explaining place value or providing a unit value distinction between the two numbers.	<p>To compare 42 with 44 and 37 with 42, student said “because 2 is less than 4” and “because 3 is less than 4”. (128, 15:47)</p> <p>To compare 37 with 42, student said “because it’s (37) the first number you come to before 40 and 60” (284, 09:36)</p> <p>To compare 5 and 7 student said “because whenever I look at the numbers, 5 is on over here and 7 is more up, that its more further” (337, 11:09)</p>



Learning Progression Reconciliation

Conceptualization of Content

Ordering

5. Composition									
Code	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
NRR.B.5.a.	Compose a number with single objects.								
NRR.B.5.b.				Compose a number with two parts.					
NRR.B.5.c.				Compose a number with three or more parts.					
NRR.B.5.d.				Compose a number with two or more parts using different number combinations .					
NRR.B.5.e.				Compose a number with two or more parts using concepts of place value .					



Developmental Appropriateness



Reconciliation: Composition

5. Composition									
Code	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
NRR.B.5.a.	Compose a number with single objects.								
NRR.B.5.b.		Compose a number with two parts.							
NRR.B.5.c.			Compose a number with three or more parts.						
NRR.B.5.d.			Compose a number with two or more parts using different number combinations .						
NRR.B.5.e.			Compose a number with two or more parts using concepts of place value .						

5. Composition										
Code	Kindergarten			Grade 1			Grade 2			
	F	B	T	F	B	T	F	B	T	
NRR.C.5.a.	Compose a quantity.									
NEW			Compose a quantity in groups of hundreds, tens, and ones.							
NRR.C.5.b.		Compose a number with two parts.								
NRR.C.5.c.		Compose a number with three or more parts.								
NRR.C.5.d.		Compose a number with two or more parts using different number combinations.								
NRR.C.5.e.		Compose a number using hundreds, tens, and ones.								





Future Activities



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Future Activities

NRR Item Models & Think Alouds
SR Cognitive Interviews



Measuring Early Mathematics Reasoning Skills

Research in
Mathematics Education



Find out more about our projects:

<https://www.smu.edu/Simmons/Research/RME>



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Thank You!

Measuring Early **M**athematics **R**easoning **S**kills
Research in **M**athematics **E**ducation

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World Changers
Shaped Here



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