Cognition and Early Childhood Numeracy: How Number Concepts are Built and Why Input Matters

Kelly S. Mix

University of Maryland

Symbol grounding

3 three

symbol grounding

What does this mean?

四 shi/yon

五 go

六 roku

七 nana

八 hachi

Do the spoken words help?

symbol grounding

一 二 三 四 五
六 七 八 九 十

How about now?

Symbol Grounding in Early Number
How are these situations different?

"one-two-three-four-five-six-seven-eight-nine-ten"

"one-two-three-four-five-six-seven-eight-nine-ten"

How about these?

“Three”

3

“one-two-three”

=

=

<

<

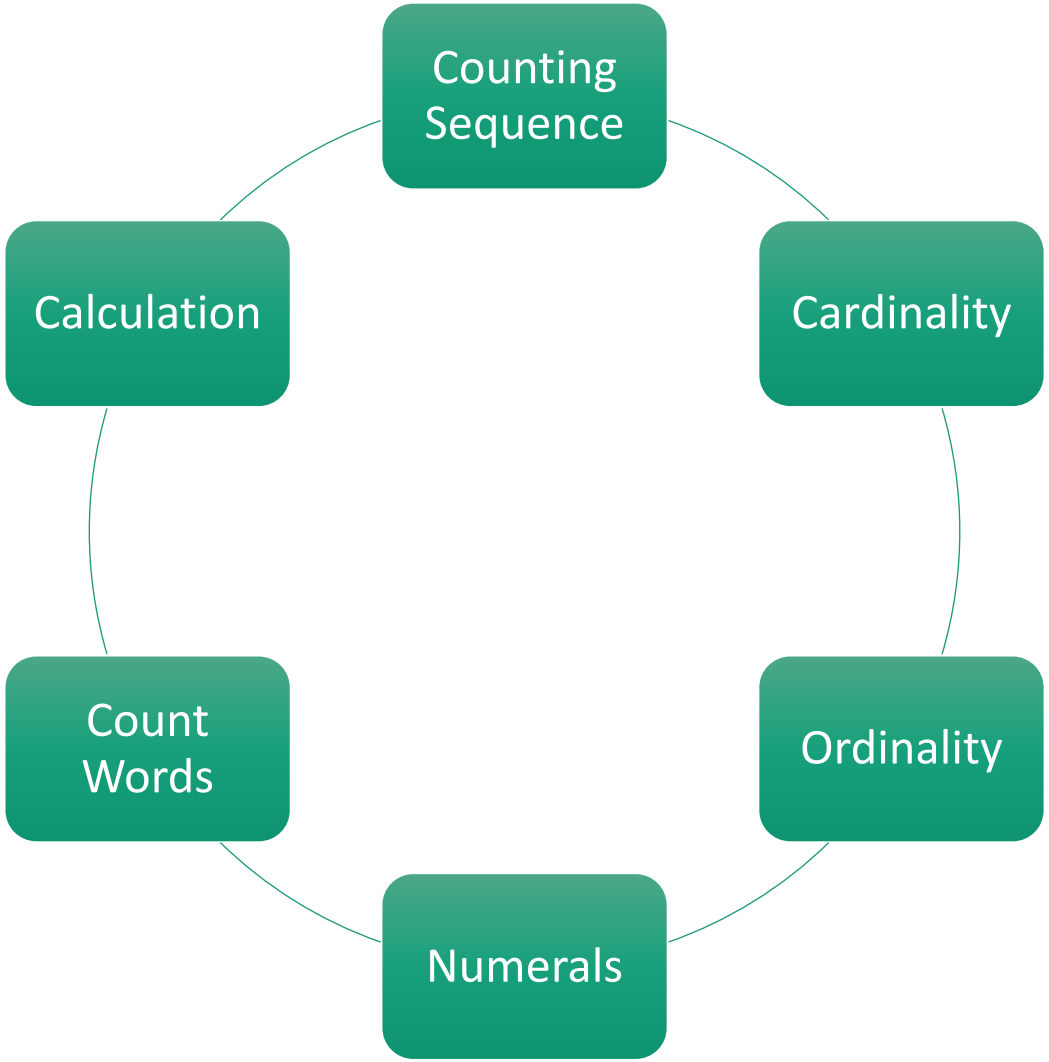

©Warren Photographic

<

<

2 < 3 < 4

Early numeracy is a system of mappings.

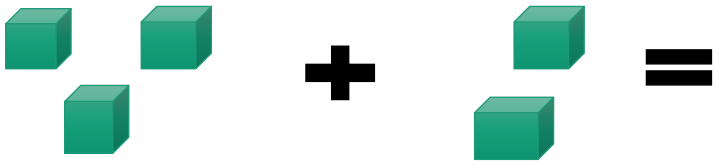


SES performance differences are symbol grounding differences.

- Children from low-SES families do well on nonverbal tasks, but not verbal tasks (Dowker, 2005; Jordan et al., 1994).

VERBAL PROBLEM
Mary picks 3 apples and John picks 2 apples.
How many apples do they have all together?

NONVERBAL PROBLEM



- Caregivers in low-SES families provide less input on counting and numeracy than those in middle-SES families (Levine et al., 2010; Jordan et al., 2006).

What cognitive processes support these mappings?

“Three”

3

“one-two-three”

Cognitive Processes

- STATISTICAL LEARNING
- STRUCTURE MAPPING
- LANGUAGE
- SPATIAL COGNITION

Cognitive Processes

STATISTICAL LEARNING

Perceiving regularities based on the likelihood of events

STATISTICAL LEARNING

Saffran, Aslin & Newport (1996)

BO **PA LU DO** SA BO TI **PA LU DO** BO RA LU TI RA DO BO MU RA **PA LU DO** TI RA SA BO PA RA

STATISTICAL LEARNING

Saffran, Aslin & Newport (1996)

BO **PA LU DO** SA BO TI **PA LU DO** BO RA LU TI RA DO BO MU RA **PA LU DO** TI RA SA BO PA RA

After 2 minutes of exposure, infants could recognize “words” like *paludo* based on the transitional probabilities

WHICH IS N? TASK

Which is four hundred twenty nine?

429

942

WHICH IS MORE? TASK

Which is more?

429

942

K-2 Percent Correct

GRADE	Which is n?	Which is more?
Kindergarten	70%	70%
1st grade	93%	89%
2nd grade	96%	97%

Mix, Prather, Smith, & Stockton (2014)

K-2 Percent Correct

GRADE	Which is n?	Which is more?
Kindergarten	70%	70%
1st grade	93%	89%
2nd grade	96%	97%

Pre-K Percent Correct

AGE	Which is n?	Which is more?
3 years	61%	57%
4 years	69%	64%
5 years	79%	75%

Mix, Prather, Smith, & Stockton (2014)

WHICH IS N? TASK

Which is four hundred twenty nine?

429

42

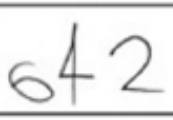
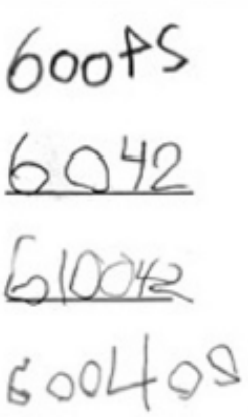
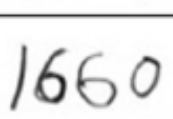
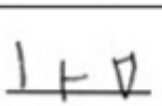
WHICH IS N? TASK

Which is four hundred twenty nine?

429

36

Overregularization: A smart error!

Classification	*Examples*
Conventional	
Expanded	
Digit Strings	
Other	

Byrge, Smith, & Mix (2014)

Percentage of Different Responses

AGE	CONVENTIONAL "642"	EXPANDED "600402"	DIGIT STRING "1662"	OTHER
4 years	1%	10%	19%	**70%**
5 years	6%	**54%**	23%	17%
6 years	33%	**48%**	14%	5%

642 = 600402

642 = 600 + 40 + 2

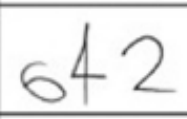
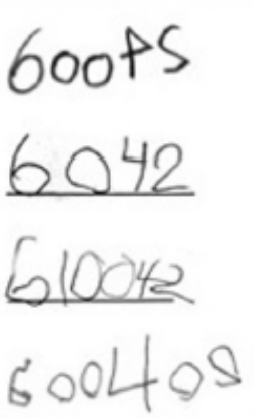
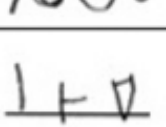
642 = 6 x 100 + 4 x 10 + 2 x 1

Byrge, Smith, & Mix (2014)

STATISTICAL LEARNING

- Children can use probabilities to learn structural relations, like numerical equivalence or place value.

- The more exposures children have, and the more regularity there is in these exposures, the easier it will be to detect patterns using statistical learning.

Classification	*Examples*
Conventional	
Expanded	
Digit Strings	
Other	

Cognitive Processes

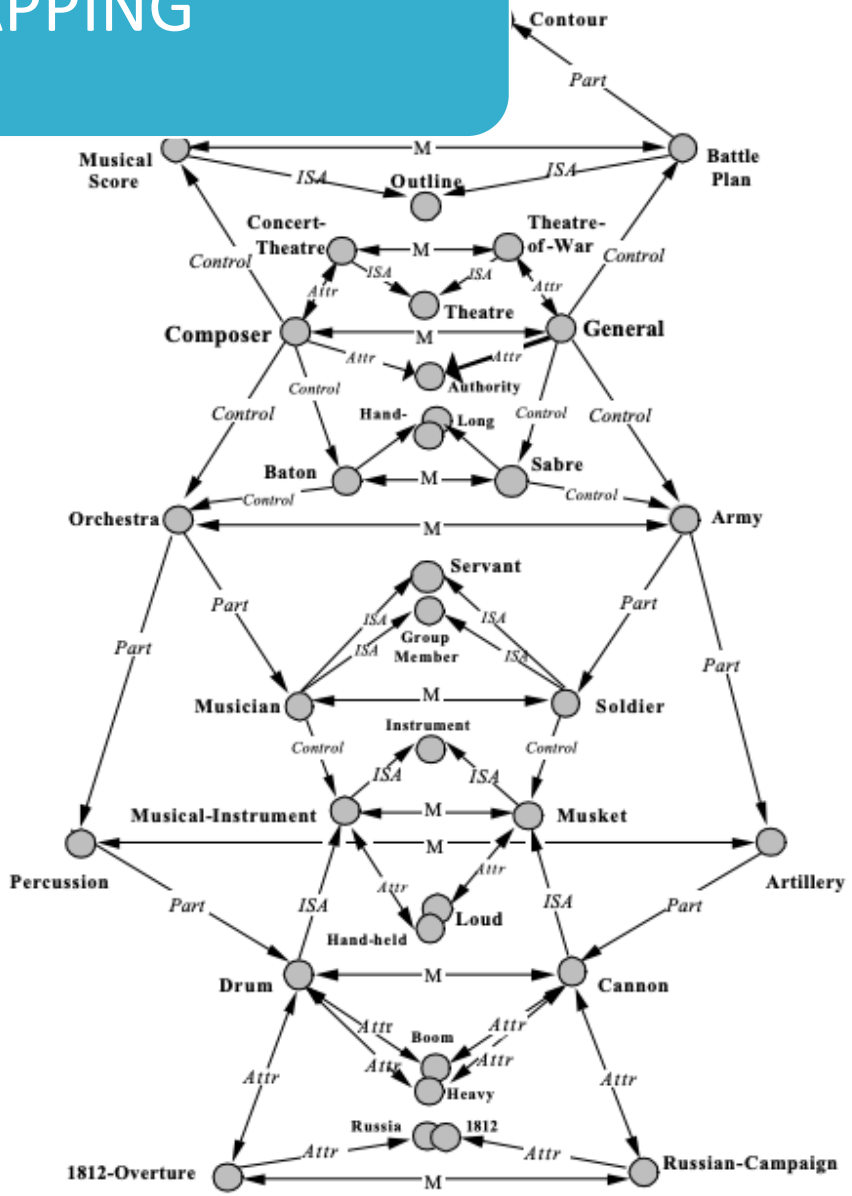
STRUCTURE MAPPING

Discovering underlying structures through the process of comparison (Gentner, 2010)

STRUCTURE MAPPING

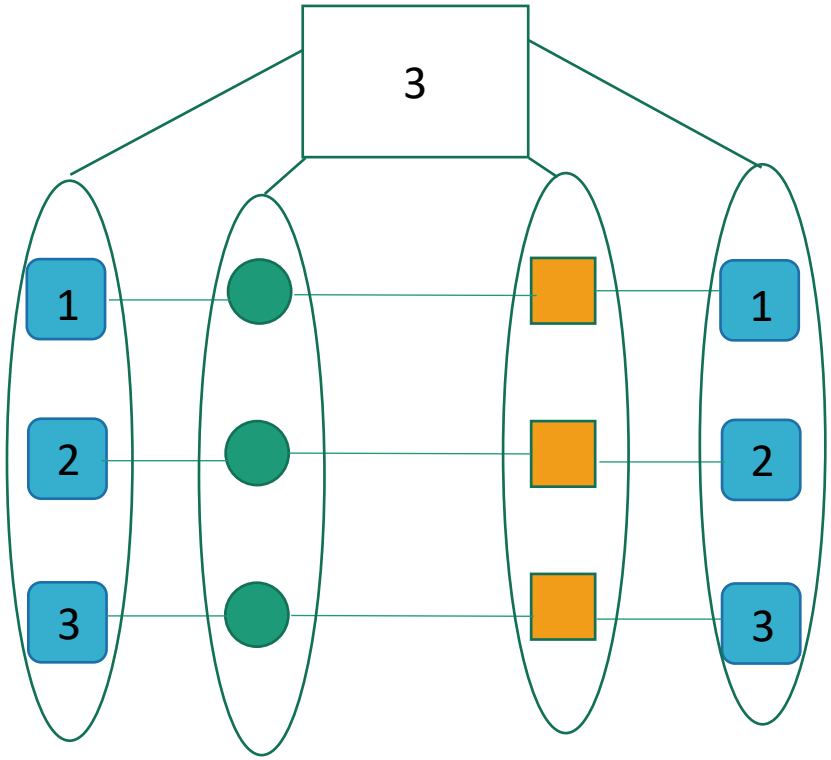
How is a composer like a general?

STRUCTURE MAPPING



(ex. from Veale & Keane, 1997)

STRUCTURE MAPPING

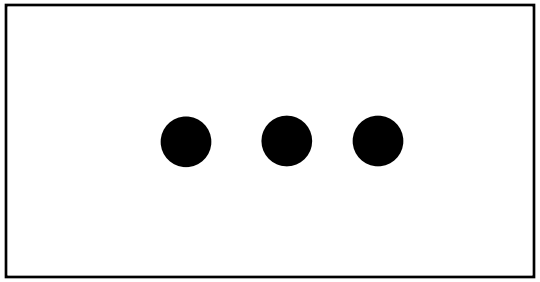


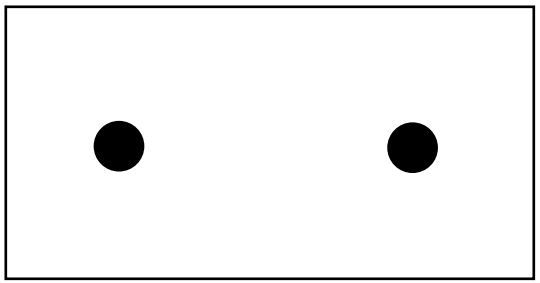
STRUCTURE MAPPING

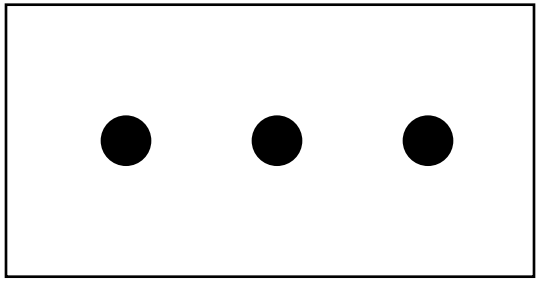
Alignable structures support comparisons and discovery of new dimensions (Kotovsky & Gentner, 1996; Sandhofer, 2003; Smith, 1989)

Mix (1999)

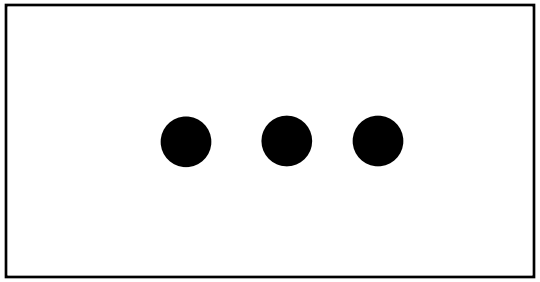
Surface similarities engage structure mapping
(Kotovsky & Gentner, 1996; Rattermann & Gentner, 1990)

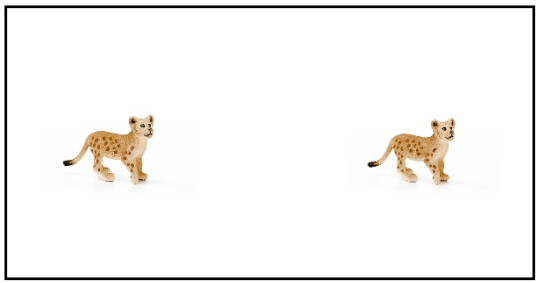


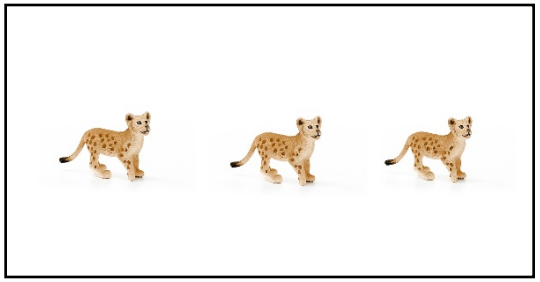




Mix (1999)







Spencer's Diary Study (Mix, 2002)

Actions engage structure mapping

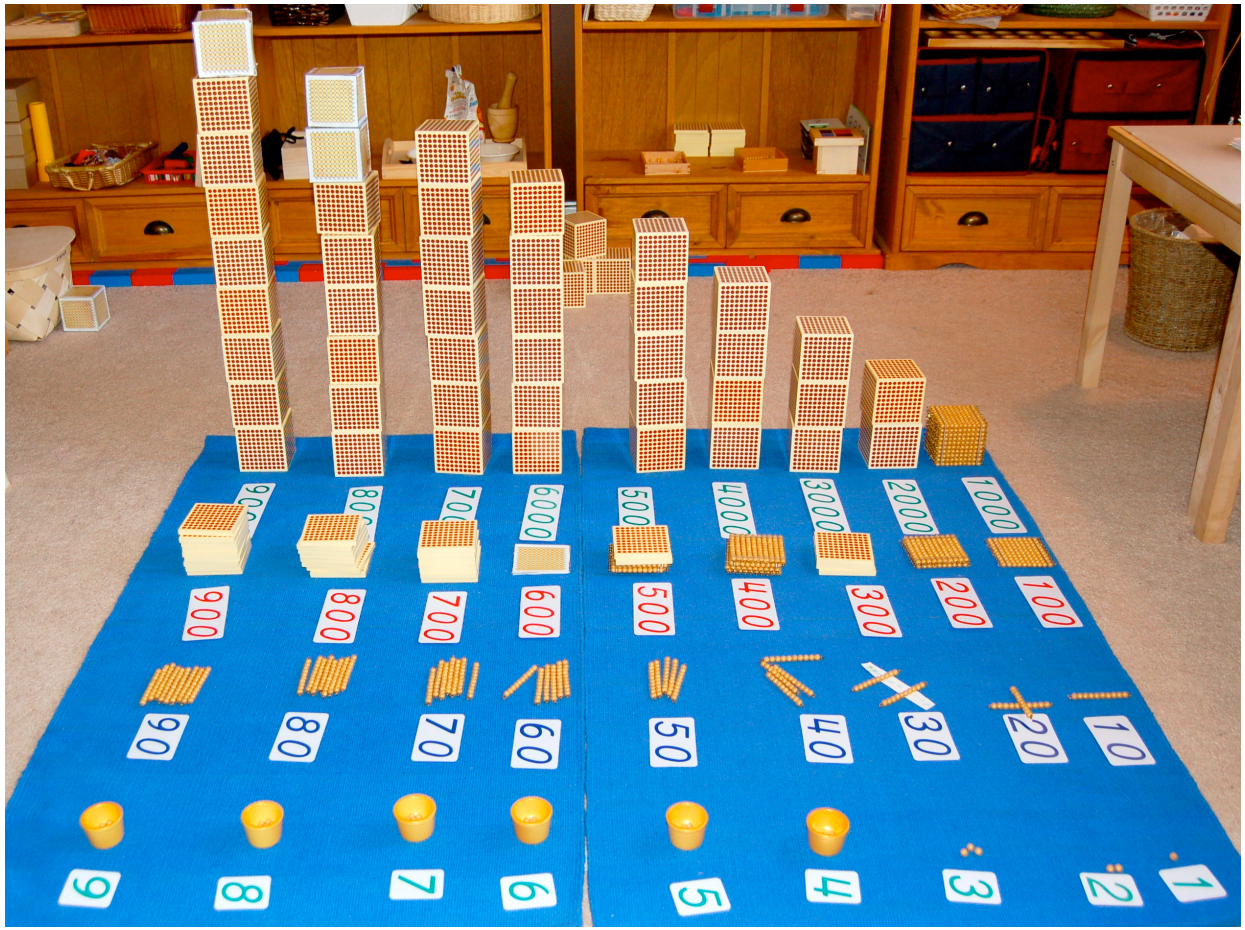
One-one correspondence toys

3.5-year-olds

Better performance on challenging matching task

(Mix, Moore, & Holcomb, 2012)

Structure in Multidigit Numbers



(Montessori)

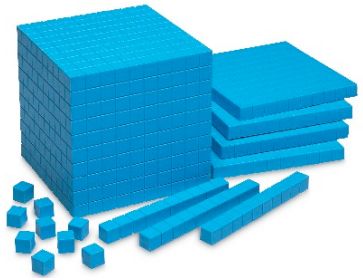
EXPERIMENT 1
(Mix, Smith, Stockton, Cheng & Barterian, 2017)

Pretest-Training-Posttest

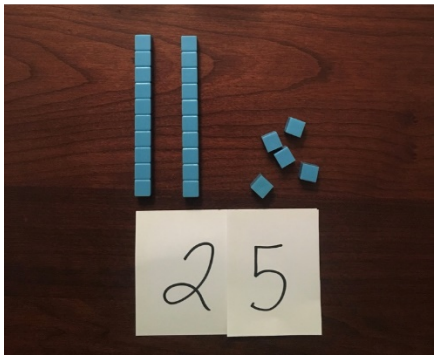
125 second grade students (age 7)
4-6 weeks of instruction

Three conditions:

a. Blocks and Symbols
b. Symbols Only
c. No Training

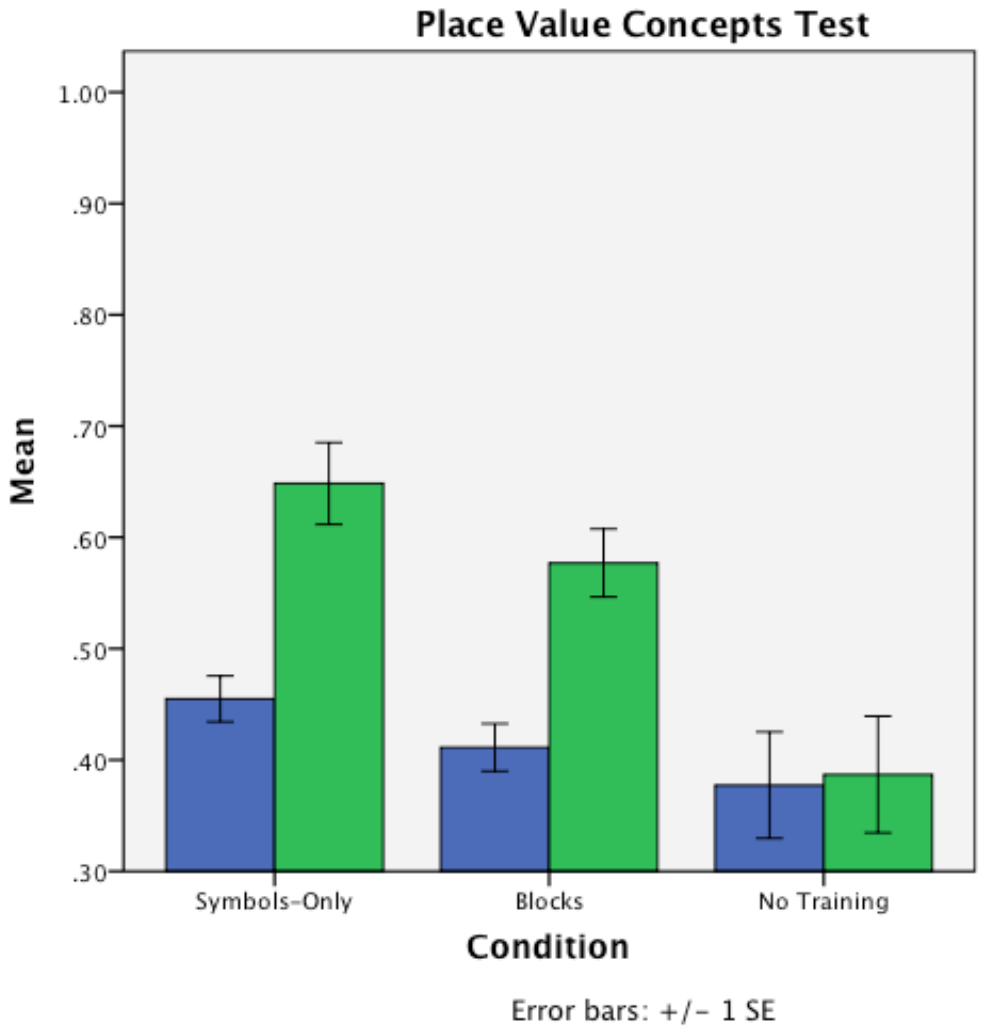


Structure Mapping is all about *comparison*



LESSON OBJECTIVE	BLOCKS	SYMBOLS ONLY
Copy the number 158	Represent with blocks	Represent with digit cards
Solve 29 + 5	Use blocks as supports	Written only

EXPERIMENT 1
(Mix, Smith, Stockton, Cheng, & Barterian, 2017)

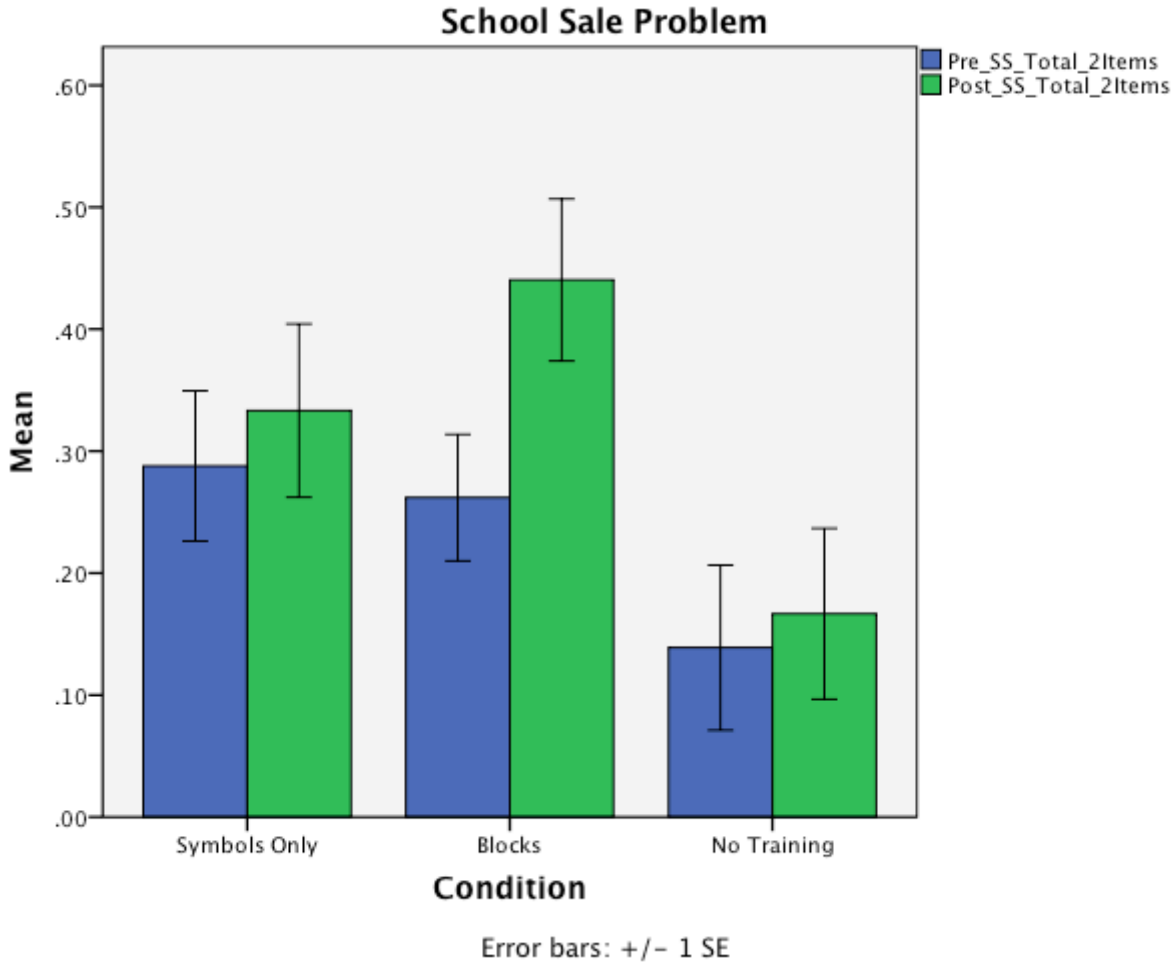


School Sale Problem

(Bednarz & Janvier, 1982)

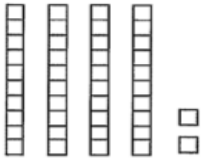
You can fit 5 candies in a roll, and 5 rolls in a box. If you start out with 38 candies, how many full boxes will you have?

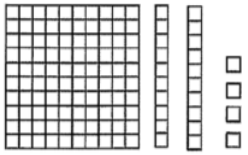
EXPERIMENT 1
(Mix, Smith, Stockton, Cheng, & Barterian, 2017)

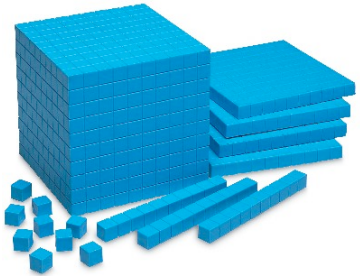


Base-10 Counting Task

Chan, Au & Tang (2014)

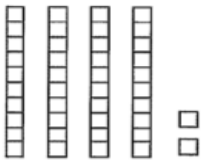


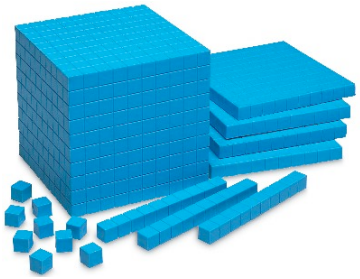


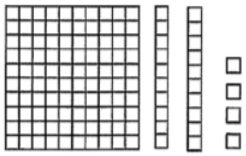


Base-10 Counting Task

Chan, Au & Tang (2014)



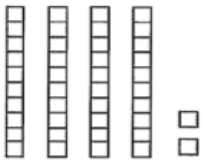


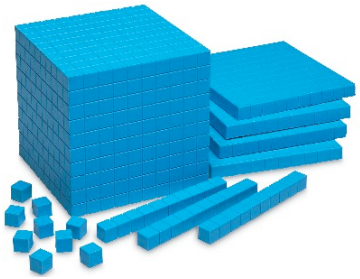


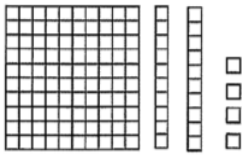
A significant predictor of 2nd grade mathematics outcomes even controlling for age, IQ, counting, calculation
(Chan et al., 2014)

Base-10 Counting Task

Chan, Au & Tang (2014)





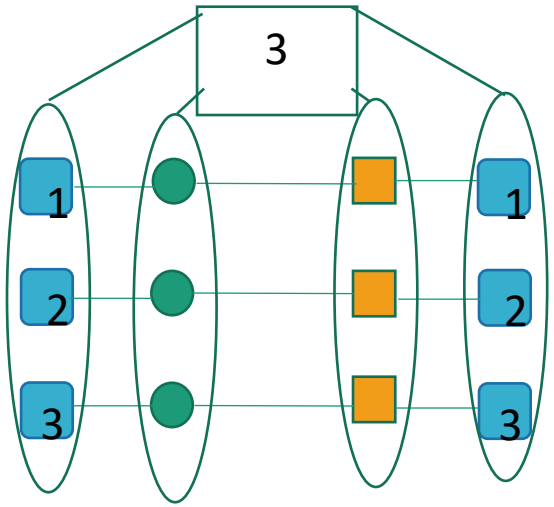


A significant predictor of 2nd grade mathematics outcomes even controlling for age, IQ, counting, calculation
(Chan et al., 2014)

The strongest predictor compared to number line, magnitude comparison & expanded notation
(Mix, Hancock, Bower, Yuan & Smith, in prep.)

Why? Structural understanding vs. approximate or holistic understanding

STRUCTURE MAPPING



- Helps learners discover underlying conceptual relations, like numerical equivalence, place value, etc.

- Comparisons and alignment highlight structure

Cognitive Processes

LANGUAGE

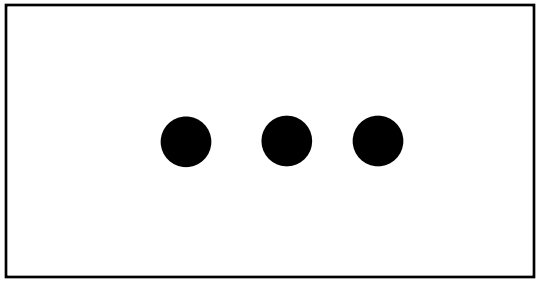
LANGUAGE

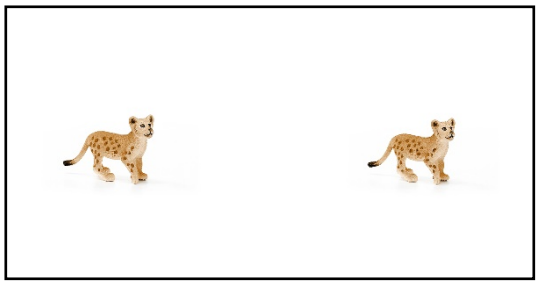
Words/labels signal similarity (Rattermann & Gentner, 1998; Sandhofer & Smith, 1999; Waxman & Markow, 1998)

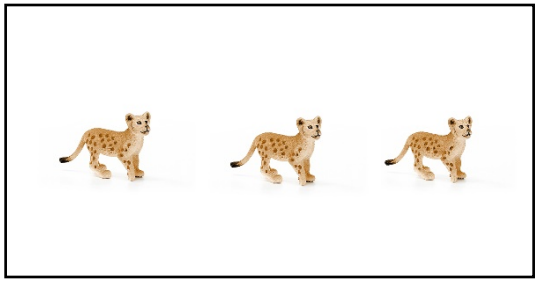
"Three dogs. Three cars. Oh, three and three! They're both three!"

=

Children recognized more challenging equivalence matches when they knew the meanings of the number words (Mix, 1999)

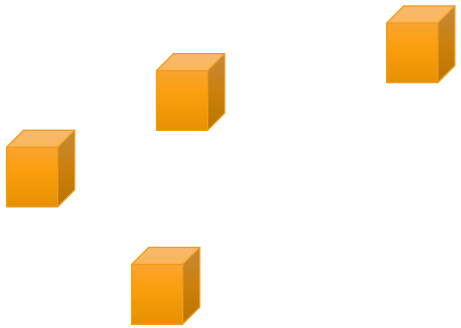






Language scaffolds mappings.

“Look! Four blocks. One-two-three-four!”

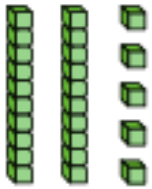


Vs. “One-two-three-four!
Four blocks.”

Mix, Sandhofer, Moore & Russell, 2011

LANGUAGE

Asian mathematics language is more transparent than English (Fuson & Kwon, 1991; Miller & Stigler, 1987; Miura et al., 1993)



“Two ten five”

Language reveals structure (Miura et al., 1999; Paik & Mix, 2003)

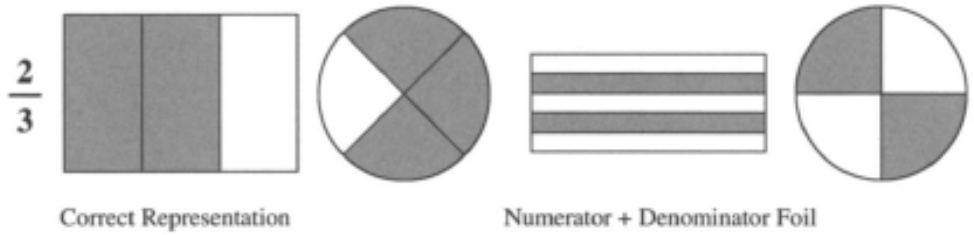


Figure 1. A sample test item—adapted from a figure used by Miura, Okamoto, Vlahovic-Stetic, Kim, and Han (1999).

"of three parts, two"

vs.

"two-thirds"

Language reveals structure (Paik & Mix, 2003)

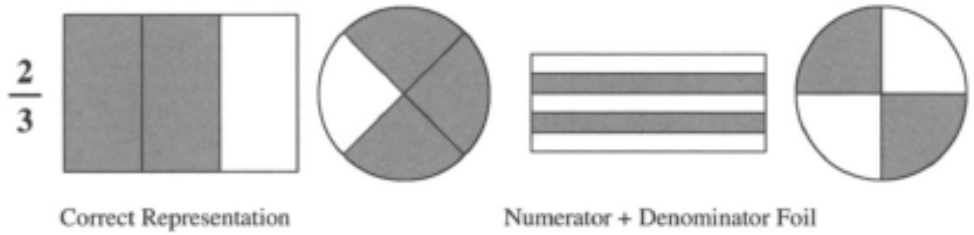


Figure 1. A sample test item—adapted from a figure used by Miura, Okamoto, Vlahovic-Stetic, Kim, and Han (1999).

"of three parts, two"	62%
vs.	
"two-thirds"	30% (chance)

1st grade students

LANGUAGE

- Common labels signal similarity
- Language can reveal structure

Cognitive Processes

SPATIAL COGNITION

SPATIAL COGNITION

- Spatial ability predicts STEM career achievement
(Shea, Lubinski & Benbow, 2001; Wai, Lubinski & Benbow, 2009)

- Spatial ability correlates with math achievement pre-K to gr. 12
(Delgado & Prieto, 2004; Gunderson, Ramirez, Beilock & Levine, 2012; Lachance & Mazzocco, 2006; Laski, Casey, Yu, & Dulaney, 2013; Lauer & Lourenco, 2016 ; Verdine, Irwin, Golinkoff, & Hirsh-Pasek, 2014)
- Spatial deficits are connected to individual differences in math, such as math LD (Geary et al., 2007) and sex differences (Casey, Nuttall & Pezaris, 1997)

Spatial Training

Mix, Levine, Cheng & Stockton (under review)

258 1st and 6th grade students

12 training sessions over 6 weeks

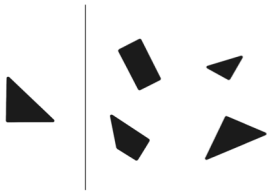
Training Conditions:

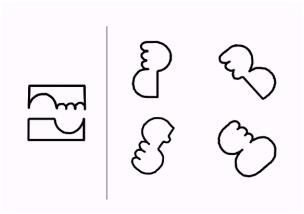
1. Spatial Training
2. No Training Control

Outcome Measures:

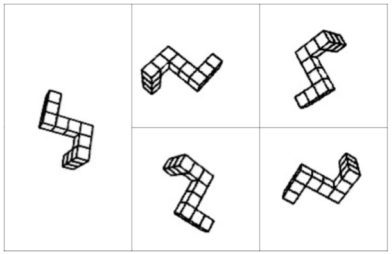
1. Word Problems
2. Place Value
3. Multi-step problems
4. Algebra/Missing Terms
5. Notational Spacing
6. Number Line Estimation

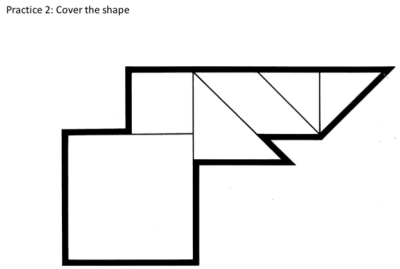
Spatial Training



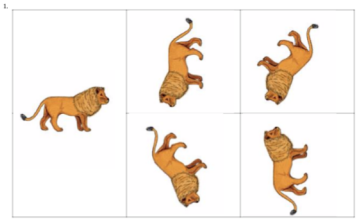


Which shape would
Make the stimulus
Into a square?



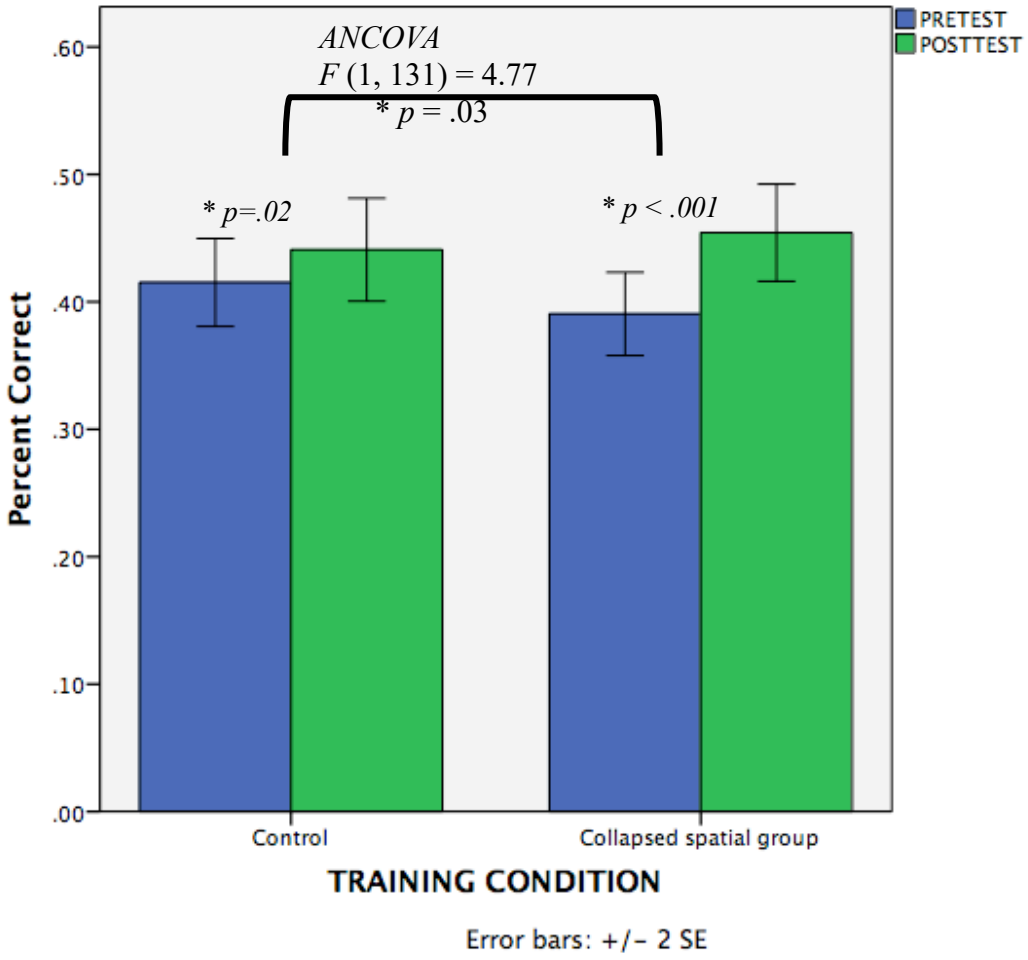


Tangrams



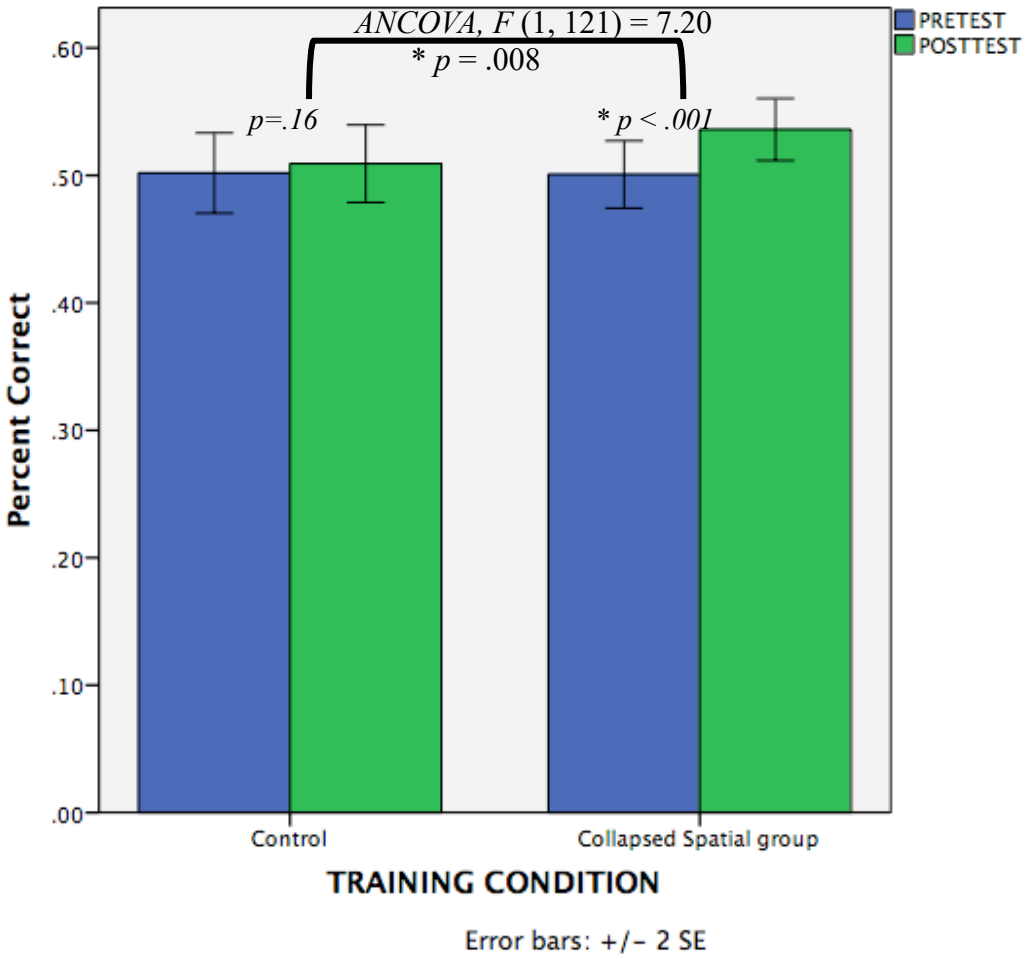
1st grade Combined Conditions

Composite Math Scores



6th grade Combined Conditions

Composite Math Scores



SPATIAL COGNITION

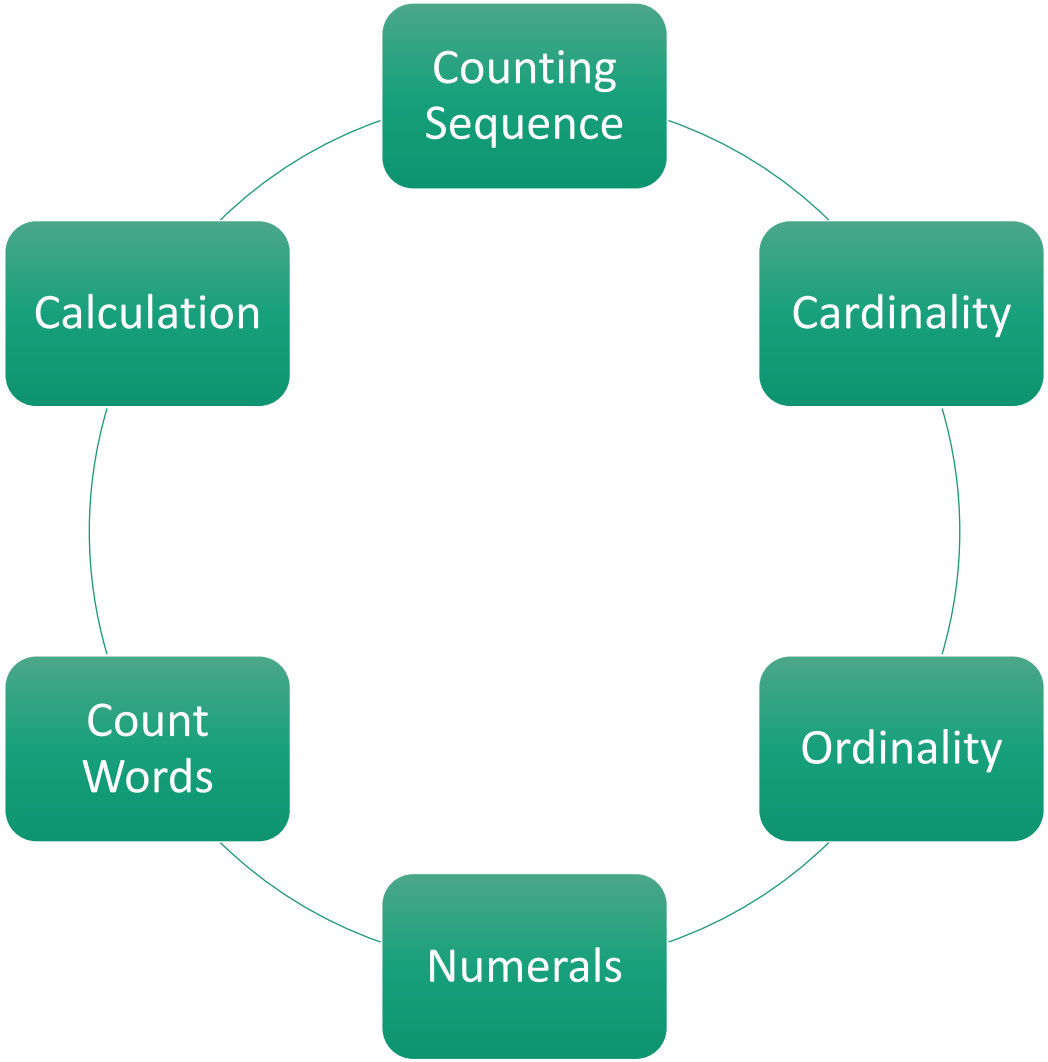
Why are space and math related? (Mix, 2019)

- Spatial skill helps problem solvers create mental models.

- In calculation, spatial skill makes it easier to read symbols or track one's place in complex procedures (e.g., long division).

symbol grounding

What does this mean?


Counting Sequence
Cardinality
Ordinality
Numerals
Count Words
Calculation

Cognitive Processes

- STATISTICAL LEARNING
- STRUCTURE MAPPING
- LANGUAGE
- SPATIAL COGNITION

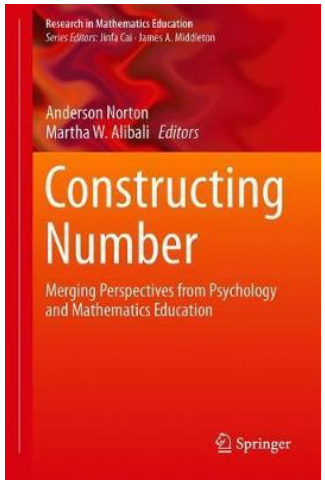
Input matters: Quantity

(Levine et al., 2010; Klibanoff et al., 2006)

- Children exposed to more math talk perform better in mathematics.
- But in 8 hours of observation, some parents talked about number/math only 3 times (range = 3 – 175).
- In a typical one-hour lesson, some teachers talked about number/math only 1 time (range = 1 – 104).

Input matters: Quality

STATISTICAL LEARNING



Dense co-occurrences
Maximum regularity

Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

35

“Thirty five. Three tens and five ones. See? Three tens. Five ones. That’s thirty-five.”

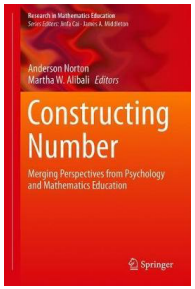
Input matters: Quality

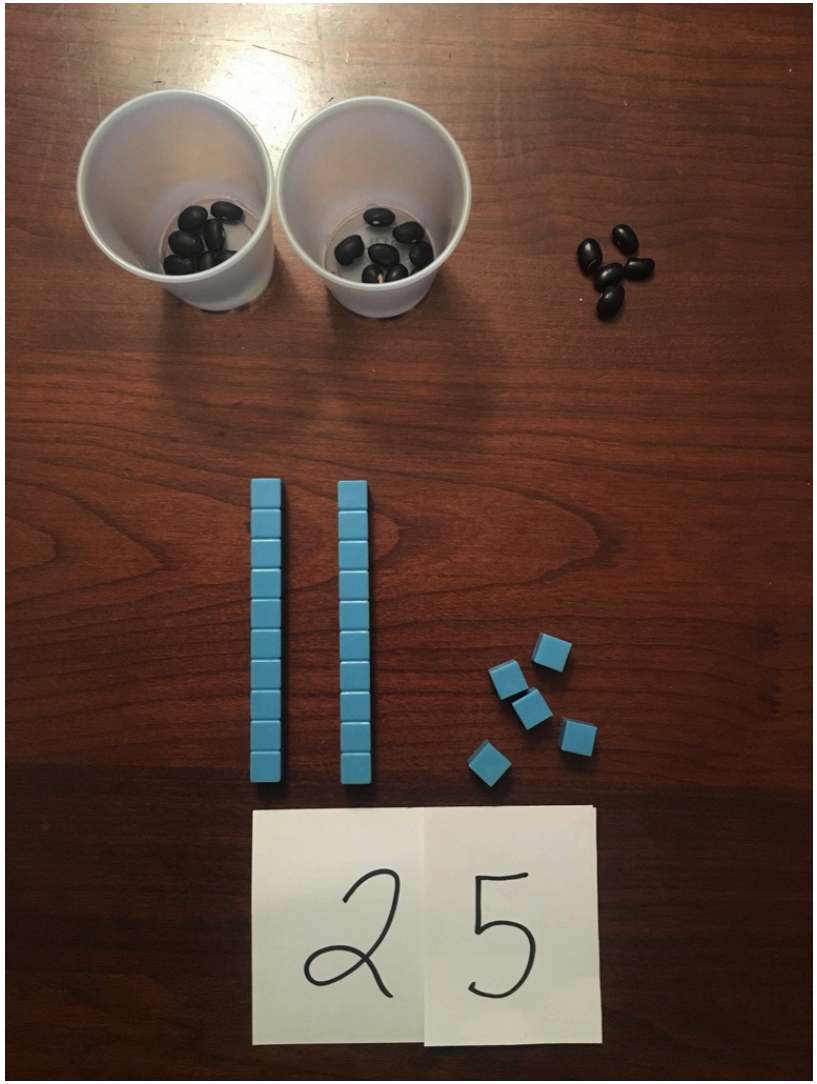
STRUCTURE MAPPING

Structural alignment via space, words, gesture

(see Mix, Crespo & Smith, 2019, for full discussion)

Progressive alignment using simple to complex mappings





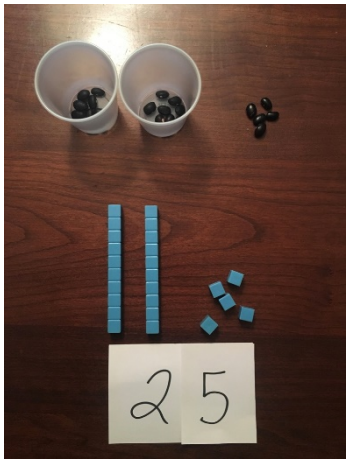
Input matters: Quality

LANGUAGE

Use words to signal similarity and encourage comparisons

"Look, three dogs, 1-2-3. And three cars! 1-2-3
Three here. Three here.
Do you think they're the same number? Let's line them up and see."

Input matters: Quality



SPATIAL COGNITION

Use space to emphasize numerical structure.

Encourage spatial activities like building with blocks or puzzles.

Encourage spatial visualization during problem solving using diagrams, objects, or mental models.

To my wonderful collaborators and funders for their generous support – Thank you!

Susan C. Levine

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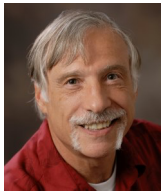

Art Baroody

Lei Yuan

Richard Prather

Sandra Crespo

Jae Paik

Greg Hancock

Linda Smith